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Application of RVT for Generation of Floor Response Spectra for the PSA Elaboration

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Abstract. The current paper considers the application of random vibration theory (RVT) for the purpose of seismic probabilistic safety analysis (SPSA) of nuclear power plants. The theory basics and assumptions are summarized, as well as an approved method for application of RVT for the probabilistic assessment of structural response to seismic excitations. The advantages and limitations of the method application are investigated through a dynamic response analysis of a WWER-1000 reactor building. The building response is analyzed by the standard flexible volume method for dynamic SSI analysis in the frequency domain. Two approaches are used for the definition of the input motion – a set of accelerograms and a mean uniform hazard response spectrum. The resultant floor acceleration response spectra are compared for both of the approaches and used as a verification of the conclusions made, concerning the reliability and specifics of the RVT techniques used for probabilistic assessment of structural response to seismic excitations.

Keywords: dynamic analysis, earthquake, probabilistic safety analysis, response spectra, random vibration

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1 Introduction

Two of the main steps in the elaboration of a seismic probabilistic safety analysis (SPSA) of nuclear power plants (NPPs) include seismic hazard analysis and fragility analysis of critical structures, systems and components (SSCs). For the implementation of the analysis, a probabilistic assessment of the structural response due to the stochastic motions (such as earthquakes) should be made.

Current practices requires that the SPSA must be performed with the consideration of the interaction between structure and soil (SSI) either in time or frequency domain, using direct or sub-structuring methods (ASCE/SEI 4-16 [1], ASCE 4-98 [2]). The current report considers the specifics and applicability of an approach for implementation of the sub-structuring method for SSI analysis, based on the random vibration theory (RVT). This method is a reliable alternative to the traditional approach, which requires definition of the input motion as a site-specific time history motion. Typically, a set of synthetic or modified recorded ground motion histories is generated, that covers a number of code requirements for compatibility with the site-specific target response spectrum (ASCE/SEI 43-05 [3]). The approach includes execution of a series of deterministic dynamic analyses and statistical processing of the resultant structural response parameters (accelerations, displacements, stresses, etc.). A crucial step is the appropriate selection of seed time histories for matching the design response spectrum, as well as the number of time histories, sufficient to obtain a stable mean response. Several studies indicate that this approach results in mean of maximum values of the response parameters that vary in large ranges (around 30%), [4]. The observed variance is attributed precisely to the incomplete coverage of the whole range of parameters of the design motion by the generated set of accelerograms.

The current report is focused on an alternative approach for analysis, which reduce the above-mentioned shortages of time histories. The approach applies the assumptions of the RVT and ensures a stable mean of maximum response estimation as well as a significant reduction in time for generation of input motions. A number of researchers (Deng, Ostadan, and Der Kiureghian) report effective application of the approach in the SSI analysis of NNPs [4–7]. A numerical procedure for application of RVT for the purpose of SSI analysis is defined and implemented in the SASSI 2015 computer code for analysis of SSI, [6, 7].

The specifics, advantages and limitations in the application of the RVT techniques for SSI analysis of NPPs are summarized in the report. A numerical example, presented in the text below, is elaborated for verification of the conclusions made.

2 Theory Background

Resistance of building structures and facilities to random dynamic phenomena has been a crucial task of structural engineers from the dawn of structural engineering until now. Most of the natural events such as earthquakes, winds, ocean waves, etc. have random dynamic nature. Scientific efforts are focused on establishing a reliable analytical definition of the random motion of a structure, excited by a random input and reliable methods for their application in the structural analysis.

The basis of the mathematical theory of random vibrations (RVT) are laid with the work of Einstein, [8], who developed equations, governing the distribution of motions of a Brownian particle, suspended in a fluid. In mechanical point of view, the solution of Einstein concerns the vibrations of a viscous damped single degree of freedom (SDOF) system, fixed at the base and excited by white noise.

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After the basis of random vibration analysis have been laid, the theory undergoes rapid development, leading to implementation in various scientific and practical fields, including seismic engineering. Some of the achievements, leading to the definition of reliable numerical expressions of random processes, applicable to structural analyses are related with the name of Crandall, [9] and the definition of the equation of motion of an SDOF system, fixed elastically and subjected to a random stationary excitation of white noise.

The theory is further developed by the establishment of relations, crucial for the solution and definition of random vibration problems in structural engineering. For example, the definitions of spectral density and the description of the excitation and response of the SDOF system in terms of their moments (averages), [10–12].

The abovementioned studies describe the basics in the application of RVT in numerical analyses of building structures. The further development of RVT is directed to the enhancement of the interpretation and modelling of the structural response and the random dynamic motions, such as earthquakes, innovations of the techniques for analysis, specification of nonstationary random processes, random vibration of random structures and many others.

3 Application of RVT to SPSA of NPP

RVT techniques can be used for obtaining statistical mean estimates of the extreme frequency-dependent structural response through a single analysis. This is the reason RVT turns to be a potentially powerful tool for execution of a SSI analysis that can provide fast and accurate estimates of the structural response quantities, needed for the safety assessment of nuclear power plants.

The application of the RVT in seismic engineering is related with the study of Der Kiureghian [5, 13] and Davenport [14]. The approach, based on RVT used in the SSI analysis in the frequency domain, includes the following basic stages:

1. *Definition of the input motion*

In random vibration analysis of structures, subjected to stationary excitations it is common that the input motion is defined as a power spectral density (PSD) function. However, in earthquake engineering it is convenient that a site-specific mean acceleration response spectrum is used for representation of the input motion.

2. *Conversion of the input motion into a power spectral density (PSD) function*

Following the study of Der Kiureghian [5, 13], the PSD function of the stationary response of a linear system is the product of the system transfer function and the PSD of the input process (Eq. 1).

$$S_\mathrm{d}(\omega) = H^2(\omega) S_\mathrm{a}(\omega)\,, \tag{1}$$

where $S_\mathrm{d}(\omega)$ is the relative displacement power spectral density; $S_\mathrm{a}(\omega)$ is the acceleration power spectral density function; $H^2(\omega)$ is the transfer function between the displacement response and the absolute acceleration input of an SDOF oscillator with frequency ω_0 and damping ξ, defined by the following equation:

$$H^2(\omega) = \frac{1}{(\omega_0^2 - \omega^2) + 4\xi^2\omega_0^2\omega^2}\,. \tag{2}$$

The procedure proposed for generation of a PSD function, compatible with a specified acceleration response spectrum is iterative. It takes advantage of the narrow-band properties of lightly damped, SDOF oscillator transfer functions (TF). Crucial stage is the determination of the statistical mean value of the maximum response of the oscillator to a zero-mean Gaussian excitation. The solution of the problem is based on the theory of extreme value statistics and the Parseval's theorem, [5, 13–15]. The cumulative distribution of the peak absolute response over a duration t has been defined as a function of the parameters for mean zero-crossing rate of the process, ν and the shape factor for the response PSD, d. Both parameters are expressed as functions of the first three moments of the response PSD function. Due to the limitations of the report

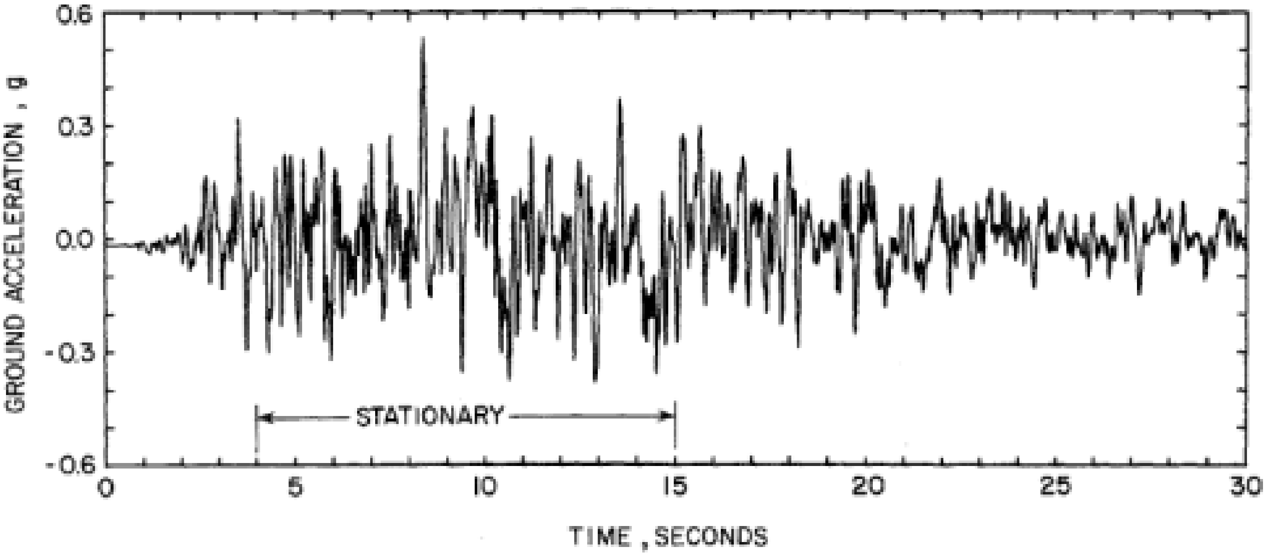

Figure 1. Sample of simulated earthquake ground motion and stationary phase of the motion (source: [13]).

length, the reader is referred to [5] for numerical expressions of the parameters.

The mean and standard deviation of the peak absolute response over a duration t (R_t) may be obtained in general as $\bar{R}_\tau = p\sigma_R$ and $\sigma_{R\tau} = q\sigma_R$, respectively, where p and q are peak factors, defined as functions of the three spectral moments and the duration t. Noticing that $\lambda_0 = \sigma_R^2$, then the mean of maximum response of the oscillator is:

$$\bar{R}_\tau = p\sqrt{\lambda_0}\,, \tag{3}$$

where λ_0 is the zero moment of the response PSD, [5].

Concerning the validity of the theory assumptions for earthquake excitations, in the above formulations, t is taken as the strong motion duration of the earthquake, which is assumed nearly stationary and the peak response usually occurs during this phase (Figure 1). Definitions and equations for the estimation of the strong motion duration are proposed by the current codes (ASCE/SEI 4-16 [1], ASCE/SEI 4-98 [2]) as a function of various seismo-tectonic parameters. The assumption of a Gaussian excitation is acceptable from the point of the central limit theorem, since the earthquake ground motion is an accumulation of a large number of randomly arriving pulses.

The applicable iterative procedure for conversion of the acceleration response spectrum into a PSD function uses the formulations above and includes the following steps:

- Conversion of the acceleration response spectrum $RSa(\omega)$ to a relative displacement response spectrum $RSd(\omega)$;
- Assumption of any acceleration PSD function $Sa,0(\omega)$;
- Calculation of the mean of the maximum relative displacement response for all the frequencies, defining the response spectrum with the assumed $Sa,0(\omega)$ and the relations given above. This represents the new relative displacement response spectrum $RSd,1(\omega)$;
- Calculation of the ratio $R(\omega) = RSd(\omega)/dRSd,1(\omega)$;
- Correction of the assumed acceleration PSD function $Sa,0(\omega)$ by the square of the ratio $R(\omega)$ and calculation of a new acceleration PSD function $Sa,1(\omega)$.

Iterations are repeated until a correction factor with a specific value is reached (usually 5–10%).

3. *Calculation of the mean of the maximum structural response*

The calculation of the mean of the maximum response for different structural points requires the acceleration PSD function, $Sa(\omega)$ of the input acceleration and the transfer function between the acceleration input and any desired response (displacement, stress, acceleration, etc.), $Hr(\omega)$. The transfer functions are computed through a standard SSI analysis frequency-domain method. The calculations use the defined relations above.

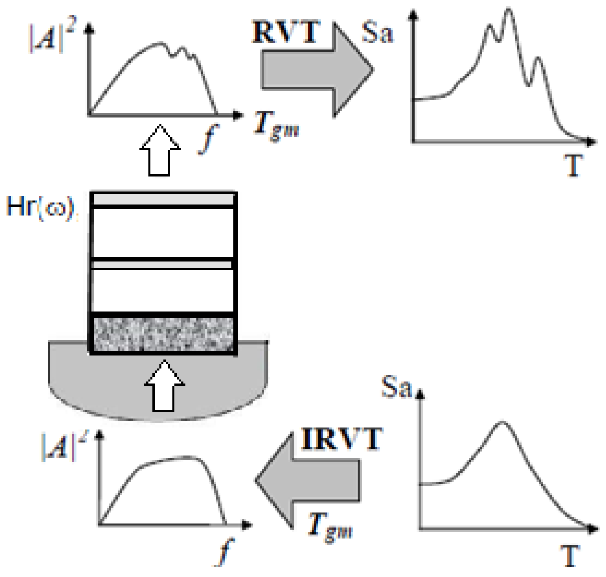

Figure 2. Schematic of the RVT approach for SPRA of building structures.

Figure 2 illustrates the schematic of the exposed RVT approach for the SPRA of buildings.

The assumptions of the RVT techniques for seismic response analysis of building structures lead to some conclusions for the advantages and limitations of their application:

- RVT proposes a convenient form of the input motion representation;
- Reduced time for structural analysis and preparation of input data in comparison with the acceleration time history approach;
- Reliable results for building structures with a pronounced fundamental natural mode of vibrations;
- Reliable results for sites with prevailing seismicity that excludes short duration pulse-like earthquakes;
- Limited to equivalent-linear structural systems;
- Results are presented as maximum of extreme response quantities for each frequency and different percentiles. Time-histories of response quantities cannot be obtained.

4 Numerical Example

The applicability of the exposed RVT techniques for the probabilistic safety analysis of NPPs is verified by a numerical example. The example considers the structural response of a WWER-1000 reactor building to seismic excitations, including the SSI.

This type of building is symmetric and its free vibrations are dominated by a single pronounced mode at each of the two main horizontal directions. For a fixed based finite element model of the building, around 50% of the structural mass is activated in the first mode of vibrations and around 20% in the fourth (Figure 3). Nevertheless, for the SSI finite element model, the fundamental structural response is dominated by the soil response. This is

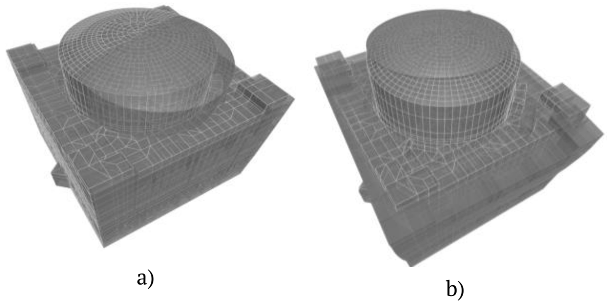

Figure 3. Dominant modes of free vibrations in main direction X for the examined WWER-1000 reactor building: a) 1-st mode ($f = 4.81$ Hz), b) 4-th mode ($f = 6.59$ Hz).

expressed by a shift of the first dominant frequency to 1.7-2.0 Hz for both of the main horizontal directions. This effect can be noticed in the floor response spectra, illustrated in Figure 4.

A finite element model of the complex of the building structure and the soil domain is generated. For the purpose of modelling and analysis, the software environment of SASSI2015 is used. The program is validated through natural experiments and approved analytic solutions, and has a vast application for the nuclear industry projects. The functionalities of the code allow for execution of linear dynamic analysis in the frequency domain with the consideration of the soil-structure interaction (SSI), using the flexible volume method. The non-linear response of the soil is considered by effective strength parameters, consistent with the strains, developed in the soil domain.

A comparison of the probabilistic estimation of the computational model response is made, when two approaches for definition of the input motion are applied: using accelerograms, consistent with the uniform hazard free field response spectrum and using the uniform hazard free field response spectrum itself.

A set of thirty three-component acceleration time histories is generated for the first case using the Latin hypercube sampling (LHS) method for generating random samples of parameter values. The set includes synthetic and modified recorded earthquake accelerograms. The time histories are generated for seed accelerograms, recorded at the site of interest or at sites with similar geological and seismological characteristics. The requirements of ASCE/SEI 43-05 for matching the uniform hazard response spectrum are met, as demonstrated in Figure 5.

The solution in the first case includes execution of a series of deterministic dynamic analyses and a statistical processing of the resultant structural response parameters. The solutions to the equations of motion in SASSI2015, [6, 7] are carried out for steady state harmonic motions for each considered frequency. Therefore, for the calculation of the responses due to transient motion, Fast Fourier Transform is performed first. In this case of analysis, except for the probabilistic definition of the input motion, several structural parameters are also probabilistically defined, including soil properties, soil layer thickness and structural materials properties.

In the second case of analysis, the statistical mean values of extremes of structural response parameters are estimated using the assumptions of the random vibration theory. For this analysis the structural and soil properties are defined as their estimated mean values.

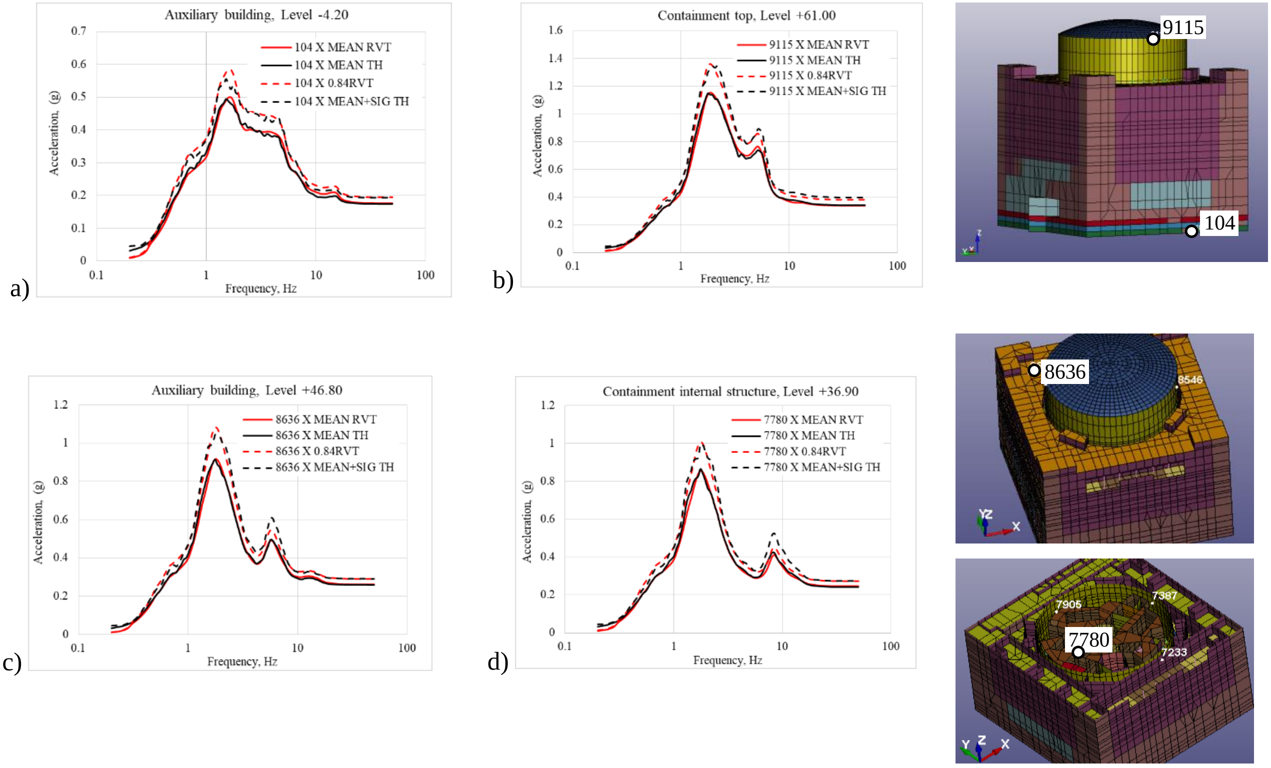

Figure 4. Comparison of floor acceleration response spectra, generated by analysis using accelerograms and RVT: a) at the foundation level (node 104), b) at the top of the containment (node 9115), c) at top of the auxiliary building (node 8636), d) plates of containment internal structure (node 7780).

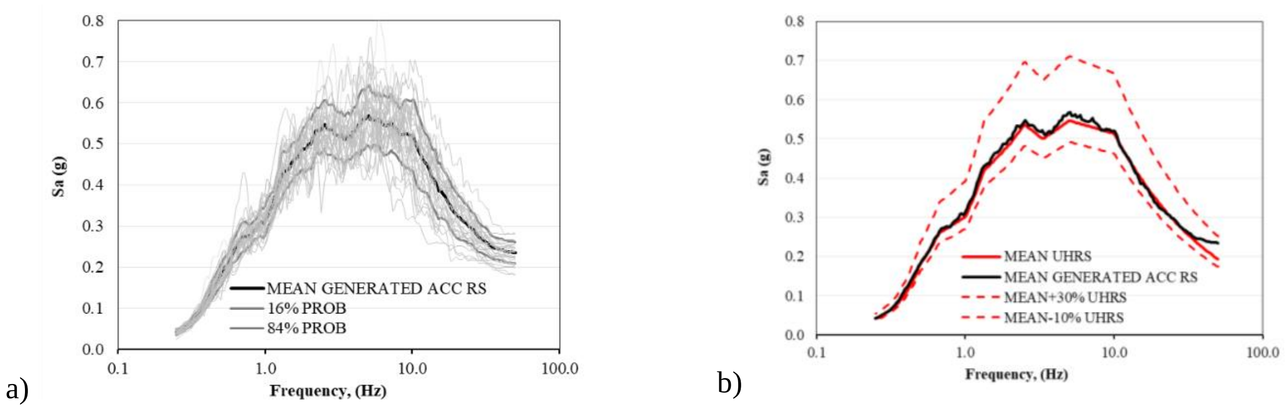

Figure 5. Acceleration response spectra (ARS) (5% damping): a) of thirty generated time histories, b) mean of the thirty ARSs and uniform hazard ARS (UHRS).

In the developed numerical example, the structural response is evaluated by the floor acceleration response spectra, calculated at 30 nodes of the structural model, located at various levels and structural elements. The resultant mean and mean plus one standard deviation (84 percentile) acceleration response spectra (5% damping), computed by both of the approaches are compared and analyzed. The comparison is illustrated for four of the analyzed nodes in Figure 4.

The following observations are made from the comparison:

- The structural response is practically identical for both of the applied approaches;
- The RVT procedure accurately matches the peak spectral acceleration and frequency, calculated by time histories.
- In case a second spectral peak occurs, a difference between spectral accelerations around 7-8% is observed for the frequencies of the rising slope of the second peak (Figure 4d). The second peak spectral frequency and acceleration also match well.
- The absolute acceleration (calculated at frequency of 30 Hz) is also well matched by the approaches used. The observed difference, averaged over the thirty analyzed nodes is 0.68% and the maximum difference is 2.31%;
- For the very low frequencies (below 0,2 Hz) the difference is significant and exceeds 50%.

The represented numerical example is indicative for the correspondence of structural response, estimated by the time history and RVT approaches for analysis. The maximum difference between the responses, assessed by both of the approaches (around 7–8%) appears in case a second dominant frequency occurs and is observed in the range of the rising slope of the spectral peak. In the current example, this is the frequency range of 5–8 Hz. Although, the reported difference is not significant, it indicates that for structures with more than one dominant frequencies, the RVT techniques may not capture precisely the structural response at higher frequencies. The large difference, observed in the very low frequency range can be explained with the theory assumptions. However, this range (below 0.2 Hz) is not of engineering interest for the considered type of building. As a conclusion, the good agreement between the resultant acceleration floor response spectra can be explained with the predictive structural behavior, dominated by a single pronounced frequency.

5 Conclusions

Based on the performed analysis and comparison of results, it is concluded that RVT is a reliable approach for probabilistic assessment of the dynamic response of structures with a pronounced fundamental natural mode of vibrations, such as the WWER-1000 reactor building. An advantage of the method is the significant reduction of time, needed for the analysis execution, as well as the input data preparation, compared to the approach using accelerograms as input motion. In addition, the method allows for covering the full range of the design earthquake parameters, which might be underestimated in case a finite number of accelerograms is used.

Nevertheless, the RVT approach should be applied with caution in SPSA of structures with more complex dynamic behavior and numerous significant natural modes of vibrations as well as for sites, where short-duration pulse-like accelerations are typical.

References

[1] ASCE/SEI 4-16 (2016) Seismic Analysis of Safety-Related Nuclear Structures and Commentary. American Society of Civil Engineers.

[2] ASCE 4-98 (1999) Seismic Analysis of Safety-Related Nuclear Structures and Commentary. American Society of Civil Engineers.

[3] ASCE/SEI 43-05 (2005) Seismic Design Criteria for Structures, Systems, and Components in Nuclear Facilities.

[4] Deng N., Ostadan F. (2012) Random Vibration Theory-Based Soil-Structure Interaction Analysis, 15 WCEE, Lisboa, 2012.

[5] Der Kiureghian, A. (1979) On Response of Structures to Stationary Excitation. Report No. UCB/EERC – 79/32, June 1979, Earthquake Engineering Research Center.

[6] Ostadan F., Deng N. (2019) SASSI2015. A System for Anal-

ysis of Soil-Structure Interaction, Theoretical Manual Version 1.0.

[7] Ostadan F., Deng N. (2019) SASSI2015. A System for Analysis of Soil-Structure Interaction, Users's Manual Version 1.0.

[8] Einstein, E. (1905) On the Movement of Small Particles Suspended in a Stationary Liquid Demanded by the Molecular-Kinetic Theory of Heat. *Annalen der Physik* **17** 549 (also, reprinted in Einstein, 1956).

[9] Crandall, S., Mark, W. (1963) Random Vibration in Mechanical Systems. Academic Press, New York.

[10] Ornstein, L. (1919) *Proc. Acad. Amst.* **21** 96-108.

[11] Uhlenbeck, G., Ornstein, L. (1930) On the Theory of the Brownian Motion. *Phys. Rev.* **36** 823-841. Reprinted in Wax (1954).

[12] Wiener, N. (1930) Generalized Harmonic Analysis. *Acta Math.* **55** 117-258.

[13] Der Kiureghian, A. (1980) A Response Spectrum Method for Random Vibrations. Report No. UCB/EERC – 80/15, June 1980, Earthquake Engineering Research Center.

[14] Davenport, A.G. (1964) Note on the Distribution of the Largest Value of a Random Function with Application to Gust Loading. In: Proceedings of the Institution of Civil Engineers Vol. 28, pp 187-196.

[15] Vanmarcke, E.H. (1972) Properties of Spectral Moments with Application to Random Vibration. *J. Eng. Mech.-ASCE* **98** 425-446; DOI: https://api.semanticscholar.org/CorpusID:117960386.