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Evaluation of the Neutron Flux Distribution in Two-Zone Reactor

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Abstract. The distribution of heat release in the reactor core is important for reliable and safe operation of NPPs. During reactor campaign, the core composition changes due to fuel burnup and this leads to a change in the neutron flux distribution in the core volume.

A numerical analysis of the change in the axial profile of the neutron flux in the reactor core, which divided into two parts symmetrical relative to the core centre, is presented. A one-group neutron diffusion model is used to find the axial profile of the neutron flux.

Various combinations of the material buckling values of the two parts of core are considered. The relations between the materials buckling of the core parts that ensure the achievement of criticality are obtained. The marginal distributions of the neutron flux and picking factor are analysed.

Keywords: diffusion model, two-zone reactor, neutron flux, material buckling, criticality.

1 Introduction

One of the conditions for safe operation of a nuclear reactor is a uniform or close to such distribution of heat generation in the reactor core. Local peaks of heat generation in the reactor core may threaten to damage the fuel rods, which leads to an accident. The heat release profile determined by the distribution of the neutron flux density and the concentration of fissile nuclear fuel. The heat release profile is due to neutron diffusion and the active composition, which changes during the reactor campaign as a result of the burning of nuclear fuel. Therefore, the task of mathematical description and calculation of the neutron flux density profile in the reactor core remains relevant.

With a four-year fuel cycle, both fresh fuel assemblies and those that have already worked for 1, 2, or 3 years, and therefore have different nuclear fuel burnup depths, are in the reactor core. Thus, it is possible to profile fuel enrichment along the radius of the core by choosing the optimal arrangement of fuel assemblies, which compensates for the unevenness of the radial profile of the neutron flux density. That named physical profiling and it widely used in VVER reactors.

At the same time, the idea of using axial profiling considered back in the 70s, but found application only in fast neutron reactors, for example, in [1]. Axial profiling of fuel enrichment to VVER reactors not applied up to now, since it significantly increases the cost of fuel production, but research in this direction continues [2–4]. Therefore, the peaking factor of the axial neutron flux profile is always higher than it is for the radial profile. During the campaign of the reactor, the fuel enrichment profile along the height of the core becomes uneven due to uneven fuel burnout, even if the axial physical profiling do not used at begin of reactor campaign [5]. In addition, a regulatory group of control rods that partially inserted can have a noticeable distortion effect on the axial profile of neutron flux in the core of VVER and PWR reactors [6, 7].

Changing the axial profile depending on the material composition of the parts of the reactor core is an urgent task. The following are the results of an analytical study of the axial neutron flux profile evolution based on a diffusion model of a two-zone reactor.

2 Model of a Two-Zone Reactor

Let us consider a cylindrical two-zone reactor consisting of horizontal layers symmetrically located relative to the center of reactor core. The central part is characterized by the material buckling B_1^2, and the peripheral part by B_2^2. The boundary between the central and peripheral parts of core is located at a distance $h = H_1/2$ and the total core height is $h = H_1 + 2H_2$ (Figure 1). This simplest model

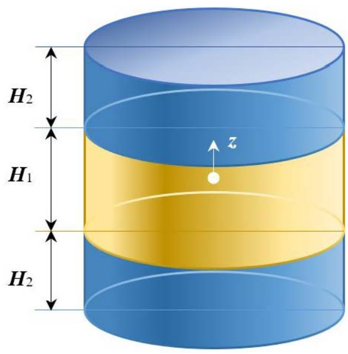


Figure 1. Model of a two-zone flat reactor with symmetry about the center of core.

allows us to investigate the change in the axial profile of the neutron flux density when changing the material composition of the core parts.

The properties of core are determined by geometric and material buckling. The axial profile of a flat two-zone reactor will depend on the position of the interface between the core parts and the ratio between the materials buckling of the core parts. The cylindrical reactor has two determining dimensions that determine the radial and axial component of the geometric buckling

$$B_g^2 = B_z^2 + B_r^2 = (\pi/H)^2 + (2.405/R)^2 \,. \qquad (1)$$

Material buckling B_m^2, which is determined by the material composition of the reactor core can be calculated

$$B_m^2 = \frac{k_\infty - 1}{M^2} \,, \qquad (2)$$

where k_∞ is the multiplication factor of the infinite sizes core; M – neutron migration.

In the critical state of the reactor, both material and geometric buckling of core become equal. It should be noted that when the multiplication factor $k_\infty < 1$, the reactor is subcritical and the material buckling accordingly becomes a negative value $B_m^2 < 0$. With respect to the two-zone reactor in critical state, two cases are possible:

- when both material buckling have a positive value, but due to the more intense burnup of nuclear fuel in the centre of core $B_2^2 > B_1^2 > 0$; or
- one of material buckling has a negative value, for example $B_2^2 > 0$ and $B_1^2 < 0$.

The diffusion equation for each part of reactor core is similar, but may differ according to the material buckling value – (3) if is positive, or (4) if is negative.

$$d^2 Z_i(z)/dz^2 + B_i^2 Z_i(z) = 0 \,; \qquad (3)$$

$$d^2 Z_i(z)/dz^2 - B_i^2 Z_i(z) = 0 \,. \qquad (4)$$

The general solution of differential equations (3) and (4) is, respectively

$$Z_i(z) = A_1 \cos(B_i z) + C_1 \sin(B_i z) \,; \qquad (5)$$

$$Z_i(z) = A_2 \cosh(B_i z) + C_2 \sinh(B_i z) \,. \qquad (6)$$

The application of boundary conditions eliminates the integration constants in equations (5) and (6). The boundary condition for the equality of neutron currents and neutron fluxes at the boundary between parts of core allows to obtain a criticality equation, which determines the combination of parameters B_1, B_2 and h, which ensures reactor criticality (B_i is the square root of the buckling B_i^2). Usually the criticality equation is transcendental, that is, it should be solved by iterations. For example, when $B_1^2 > B_2^2 > 0$, the criticality equation looks like

$$\frac{1}{B_1} cot(B_1 h) = \frac{1}{B_2} \tan(B_2[H/2 - h]) \,. \qquad (7)$$

The solutions of the criticality equation are found by specifying two of the three specified parameters, and thus find either the material buckling of one of the core parts, or the coordinate of the boundary between the core parts.

3 Combinations of the Material Buckling of Core Parts at Criticality

Variant calculations for solving the criticality equation allow us to investigate the relationship between the material parameters of the core parts when reactor in the critical state. Such a relationship between parameters B_1 and B_2 is presented in Figure 2. The dotted line corresponds to a same compositions in both parts of core when $B_1 = B_2 = \pi/H$ at different sizes of core. The three curves $B_1 = f(B_2)$ are correspond to the variations in the volume ratio of the two parts of the reactor core. At values of the ratio, $H_1/H \equiv 2h/H = 0,4;\ 0,5;\ 0,6$ the volume of the central part is 40; 50; 60% of the total volume of core.

All curves, irrespective of ratio H_1/H, intersect the dotted line at one point; this is due to the fact that only one critical value of the material buckling $B^2 = B_1^2 = B_2^2$ is possible with the same composition of both parts of the reactor core. This point of intersection may shift along the dotted line as the size H – core height changes. The maximum values of B_2 that occur at $B_1 = 0$ can be calculated by the formula

$$B_2^{\max} = \frac{\pi}{H - 2h} = \frac{1}{1 - 2h/H} \frac{\pi}{H} \,,$$

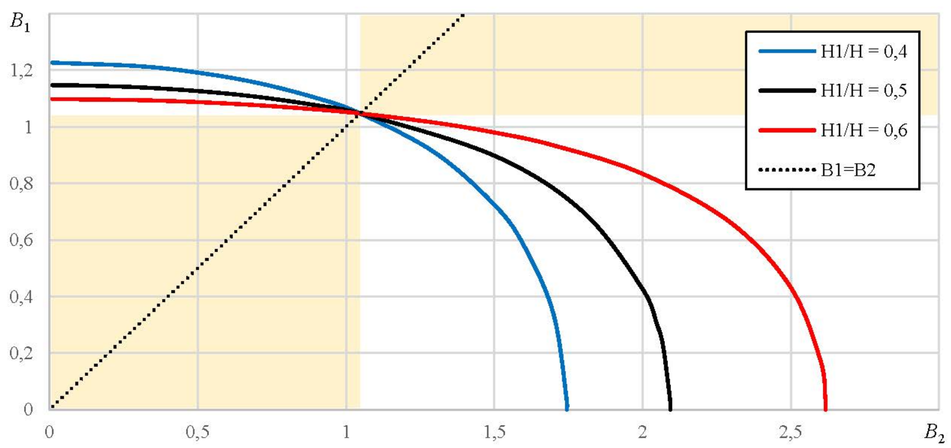


Figure 2. Dependence between the parameters of the core parts in a critical state of reactor at $B_1^2 > 0$, $B_2^2 > 0$ and different H_1/H.

and they are also scaled by H. For three variants presented in Fig. 2, $B_2^{\max}$ are equal to $1.666\pi/H$; $2\pi/H$; $2.5\pi/H$.

In Figure 2 two areas highlighted by yellow colour that correspond to combinations of parameters B_1 and B_2 values that are impossible at the critical state of the reactor core. The upper rectangular region is the region of the supercritical state of the reactor $B_1 > B_*$ and $B_2 > B_*$, and the lower is the region of the subcritical state of the reactor $B_1 < B_*$ and $B_2 < B_*$. It follows that in the critical state, the parameter for one of the two parts of core must be greater than the critical parameter B_* for the reactor as a whole, and for the other must be less than B_*. That is $B_1 > B_* > B_2$ or $B_2 > B_* > B_1$.

The combination of parameters h, $B_1 B_2$, which provides a critical state of the reactor is not symmetric with respect to the material parameters of the core parts. That is, if the critical state of the reactor reached at a certain combination of parameters $B_1 B_2$, it is not preserved if the parameters value for central and peripheral parts of core are swapped. This emphasizes that core parts of the two-zone reactor are in different boundary conditions.

The dependences presented in Figure 2 relate to the range of values of the material buckling $B_1^2 > 0$ and $B_2^2 > 0$. Thus, we can distinguish three ranges of possible combinations of material buckling of the core parts, taking into account their value – negative or positive. Three limit combinations are also possible: $B_1^2 = B_2^2$; $B_1^2 = 0$ and $B_2^2 > 0$; $B_1^2 > 0$ and $B_2^2 = 0$. When the boundary between the regions is crossed, the appearance of the neutron flux profile changes, because in each region it described by certain functions. For conditions when one of the material buckling is negative, it is also possible to obtain graphical dependences, which conditionally pass into the range of values $B_1^2 < 0$ or $B_2^2 < 0$.

Let us investigate the evolution of the axial profile of neutron flux within each region of the combination of material buckling B_1^2 and B_2^2.

4 Evolution of the Axial Profile of Neutron Flux

4.1 Both material buckling are positive $B_1^2 > 0$ and $B_2^2 > 0$

The boundary combination of conditions that both parameters are positive is when $B_1^2 = B_2^2 = (\pi/H)^2$. That is, core is conditionally divided into two parts, but is homogeneous throughout the volume. In this case, the profile described by a function

$$\Phi(z) = \Phi_0 \cos(B_z z)\,, \qquad z = 0, \dots, H/2\,, \tag{8}$$

where Φ_0 is the neutron flux in the center of the core in height $z = 0$.

The transformation of the axial profile of the neutron flux considered in the example when the core is divided into two levels by the volume of the part located symmetrically with respect to the center of the core. The height of the central part is half of the full height of the core.

The results of the calculation of the axial neutron flux profile (see Figure 3) for the variants when $B_2 > \pi.H$, $h = H/4$ and B_1^2 changes in the range $\pi/H, \dots, 0$ are presented. The profile normalized to one in the center of the core.

The lower curve in Figure 3 corresponds to the boundary variant $B_1 = B_2 = \pi/H$. To support criticality with increasing B_2 at a fixed boundary between the core parts the parameter B_1 decreases. In the extreme case, when the distribution of the flux density in the central part of the core approaches the horizontal line at $B_1^2 = 0$, and the parameter B_2 reaches the maximum possible value.

Similar calculations were performed for variants when $B_2 < \pi/H$ and presented in Figure 4, where the lower curve does correspond to the basic variant $B_1 = B_2 = \pi/H$. The profiles for the considered variants are very dense when B_2 is reduced from π/H to almost zero.

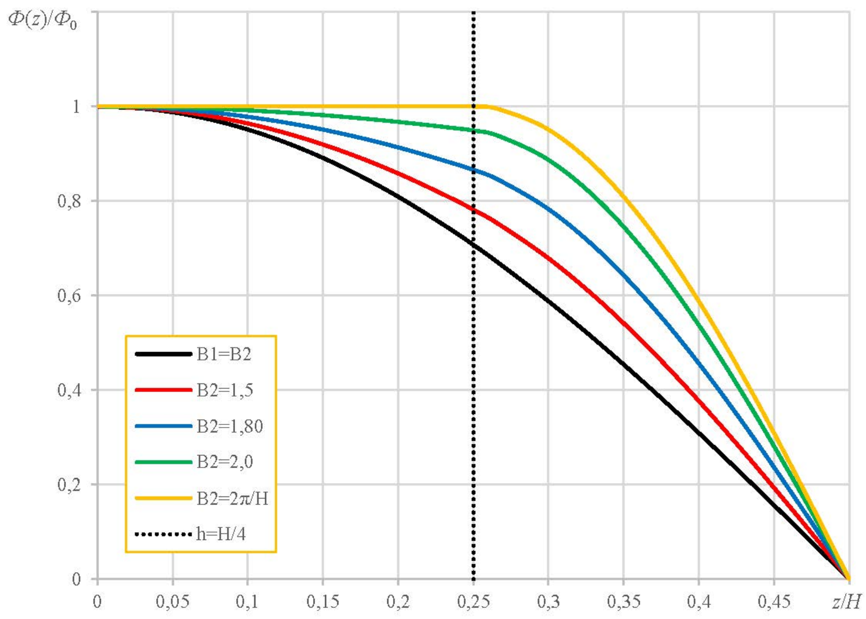


Figure 3. Transformation of the axial profile of neutron flux for $B_2^2 > B_1^2 > 0$ and $B_2 > \pi/H$.

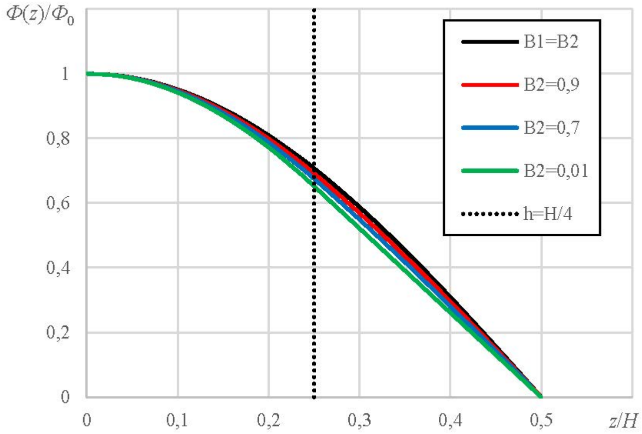


Figure 4. Transformation of the axial profile of neutron flux for $B_1^2 > B_2^2 > 0$ and $B_2 < \pi/H$.

The neutron flux profile $Z_2(z)$ in the peripheral part of core at $B_2 \to 0$ approaches the line. At the same time, in the profile of the central part becomes more uneven.

Thus, at the limit value of the material buckling of the one core part, that is equal to zero ($k_\infty = 1$), the profile of the neutron flux in this part becomes a linear dependence. We will remind that when the central part parameter $B_1 = 0$, the profile of the neutron flux in that core part $Z_1(z) = \text{const}$ (horizontal line in Figure 3).

4.2 The material buckling of the core parts are $B_1^2 < 0$ and $B_2^2 > 0$

The results of variant calculations for the conditions $B_1^2 < 0$ and $B_2^2 > 0$ are presented in Figure 5. The lower black curve corresponds to the boundary variant when $B_1 = 0$ and $B_2 = \pi/(H - 2h)$. With a further increase of B_2 to achieve criticality at a fixed boundary between the core parts, the parameter B_1 should be also increases, but $B_1^2 < 0$ becomes more negative. As in the previous variant, when both material buckling have a positive value then growth of B_2 is limited by the maximum possible value which is twice as large as the initial minimum, when $B_1 = 0$.

When B_2 approaches to this limit value, the neutron flux in the central part of core decreases many times compared to the peripheral part. The material of the core central part allegedly degenerates, becomes such that it does not affect the neutron flux, for example, as a vacuum. The evolution of the axial profile of the neutron flux in the entire range of change of the parameter B_2 can be clearly seen when the neutron flux profile is normalized to its average value, as shown in Figure 6.

When parameter of central core part approaching to $B_2^{\max} = 4\pi/H$, provided that $H_1 = H/2$, of the two core parts, in fact, only the peripheral region is represented by two symmetrically located layers of height $H/4$. The maximum value of the neutron flux profile is first located in the central core part, and with increasing of B_2 passes into the peripheral core part and gradually increases. In the boundary case when $B_2^{\max} = 4\pi/H$ the maximum value of the profile is mediocre of the peripheral layer.

For another ratio H_1/H_2 between the core parts, the boundary values of the parameters changes, but the evolution of the axial profile of the neutron flux is similar to that shown in Figure 6.

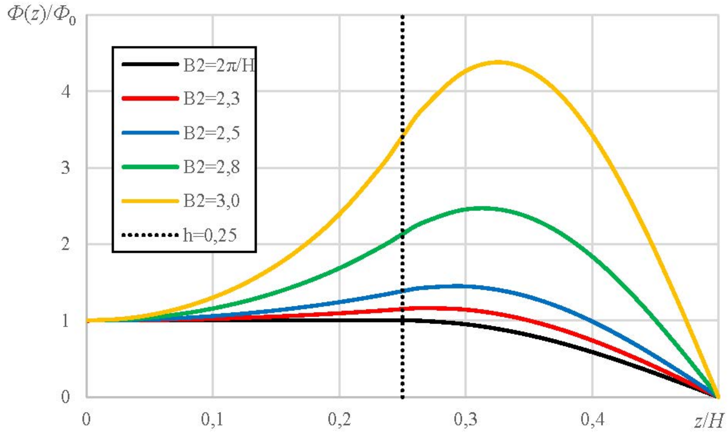


Figure 5. Transformation of the axial profile of neutron flux when $B_1^2 < 0$ and $B_2^2 > 0$.

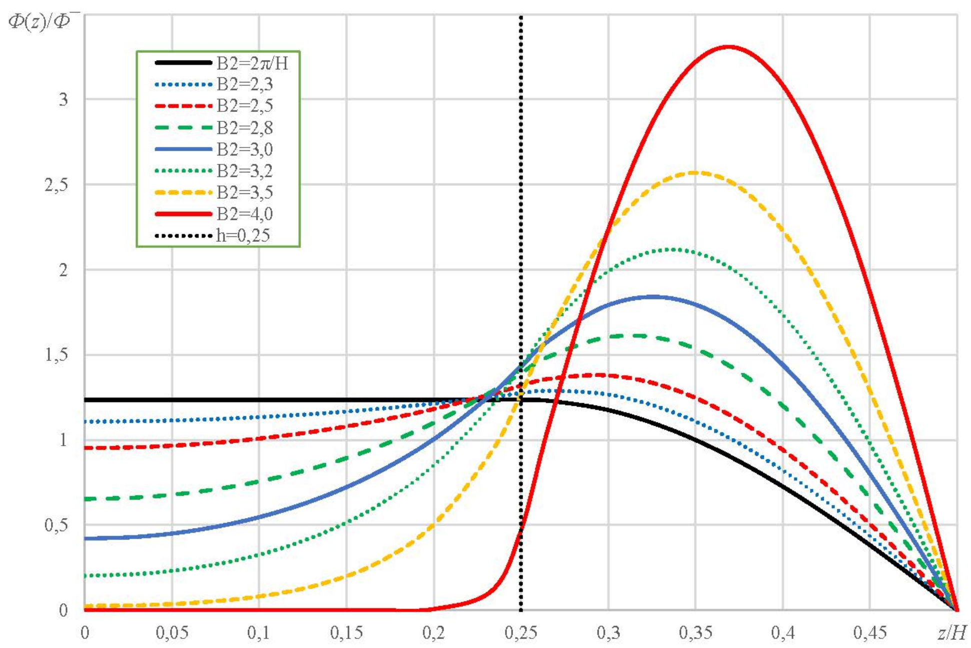


Figure 6. Transformation of the axial profile of neutron flux (normalized to the average flux value) when $B_1^2 < 0$ and $B_2^2 > 0$.

4.3 The material buckling of the core parts are $B_1^2 > 0$ and $B_2^2 < 0$

The results of variant calculations for the conditions $B_1^2 > 0$ and $B_2^2 < 0$ are presented in Figure 7. The upper curve corresponds to the boundary variant when $B_2 = 0$ and $B_1 = \pi/(H - 2h)$. With a further increase of B_1 to achieve criticality at a fixed boundary between the core parts, the parameter B_2 also increases, but $B_2^2 < 0$ becomes more negative.

When approaching to the boundary value $B_1^{\max}$, the neutron flux in the peripheral core part decreases many times in comparison with the central core part. The material of the peripheral part allegedly degenerates, becomes such that it does not affect the neutron flux density, for example, like a vacuum.

Assuming that $H_1 = H/2$ when approaching to $B_1^{\max} = 2\pi/H$, only the central region of the two core parts actually remains. The maximum value of the neutron flux density profile is always in the center of core and the profile becomes more uneven.

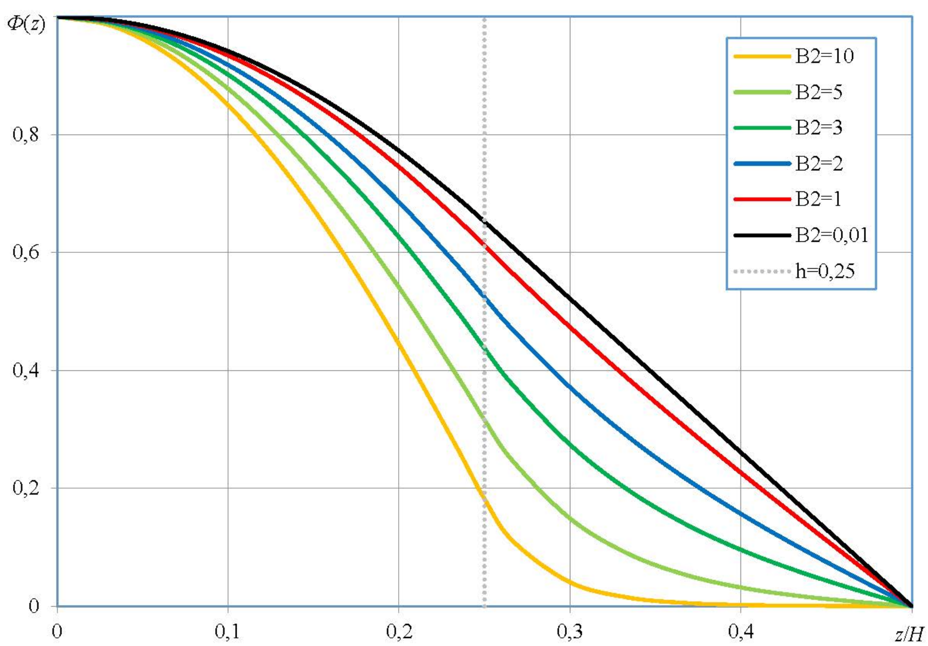


Figure 7. Transformation of the axial profile of neutron flux when $B_1^2 > 0$ and $B_2^2 < 0$.

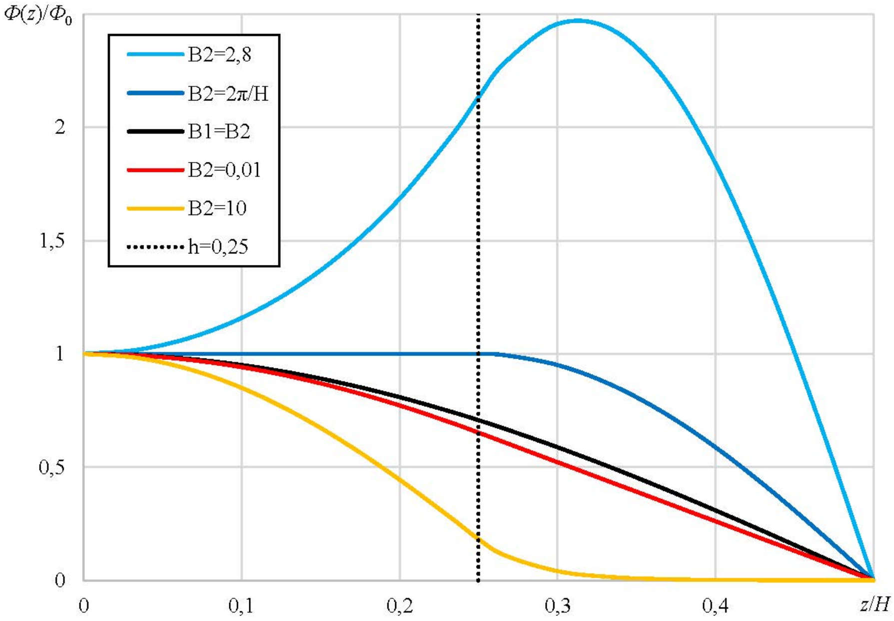


Figure 8. Evolution of the axial profile of neutron flux in the two-zone reactor.

4.4 Summary Picture of the Neutron Flux Profile Evolution

A summary picture of evolution the neutron flux profile in the two-zone reactor core presented in Figure 8. The comparative analysis was performed under the condition that the boundary between the core parts remains fixed and corresponds to the case when the central part is $H_1 = H/2$. The lower curve in Figure 8 corresponds to the values of the material buckling $B_1^2 > 0$ and $B_2^2 < 0$. As the modulus of B_2 increases, the profile remains only in the central core part and approaches to the function $\cos(z)$.

The axial profile of neutron flux marked by a red curve corresponds to the boundary combination of the material buckling values – $B_1^2 > 0$ and $B_2^2 = 0$. Under such conditions, the profile in the outer parts of core represented by a linear dependence on the core height. This curve is the boundary of the transition to the region where the values ??of the material buckling of the both core parts are positive $B_1^2 > 0$ and $B_2^2 > 0$.

The next black curve corresponds to the boundary condition when the material buckling are the same in the both core parts $B_1 = B_2 = \pi/H$. Under these conditions, the neutron flux profile along the entire core is exclusively a function of $\cos(z)$.

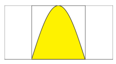
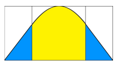
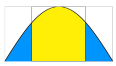
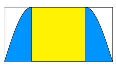
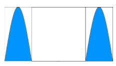
The axial profile of the neutron flux marked by a blue curve corresponds to the boundary combination of values of material buckling – $B_1^2 = 0$ and $B_2^2 > 0$. Under such conditions, the profile in the central core part represented by a linear dependence on the height and the outer core parts – by the function $\cos(z)$. This curve is the boundary of the transition to the region where the value of the material buckling of the central core part becomes negative $B_1^2 < 0$ and for the outer core parts remains positive $B_2^2 > 0$.

The last profile marked with a blue line corresponds to the values of material buckling $B_1^2 < 0$ and $B_2^2 > 0$. As the modulus of B_2 increases, the flux profile in the central core part degenerates and in the outer parts approaches to the function $\cos(z)$.

5 Picking Factor

The evolution analysis of the axial profile for the neutron flux in two-zone reactor core allows us to draw the creation conclusions about the non-uniformity of the neutron flux profile depending on the values of the material buckling of the two core parts. Comparison of the boundary values of the picking factor, which defined as the ratio of the maximum and average values of the neutron flux, are shown in Table 1.

Table 1. Picking factor of neutron flux under boundary conditions at $H_1 = H/2$

Material buckling of the two core parts	Neutron flux profile	Picking factor
$B_1^2 > 0,\ B_2^2 < 0$		π
$B_1^2 > 0,\ B_2^2 = 0$		1.6668
$B_1^2 = B_2^2,\ B_1^2 > 0,\ B_2^2 > 0$		$\pi/2 = 1.5708$
$B_1^2 = 0,\ B_2^2 > 0$		$\dfrac{2\pi}{2+\pi} = 1.2220$
$B_1^2 < 0,\ B_2^2 > 0$		π

For example, at boundary conditions $B_1^2 = B_2^2$, $B_1^2 > 0$ and $B_2^2 > 0$, the average value of the neutron flux is calculated by the equation

$$\bar{\phi} = \phi_0 \frac{1}{\pi} \int_0^{\pi} \sin(z) dz \,, \tag{9}$$

and the picking factor is

$$K = \frac{\phi_0}{\bar{\phi}} = \frac{\pi}{2}\,. \qquad (10)$$

The most close to uniform distribution of the axial profile of the neutron flux can be achieved under boundary conditions $B_1^2 = 0$ and $B_2^2 > 0$. Moreover, the given value of the picking factor can be improved by increasing the central core part, i.e. at $H_1 > H/2$.

6 Conclusions

1. Within the framework of the diffusion model of a two-zone reactor, an estimate was obtained for the axial profile evolution of the neutron flux with a change in the combination of the material buckling of the core parts, which the reactor criticality is provided.
2. Boundary combinations of material buckling of core parts are determined, the deviation from which leads to a change in the nature of the axial neutron flux profile.
3. It was found that the most close to uniform axial profile of the neutron flux can be achieved when $B_1^2 = 0$ and $B_2^2 > 0$, and if the core is divided into equal parts in height, then the picking factor is 1.222. The choice of the initial position of the boundary dividing the core into two zones significantly affects the value of the picking factor.
4. The evolution of the axial profile of the neutron flux during the life of core deserves additional consideration. When nuclear fuel burn-up, there is an interconnected change in the value of the material buckling of the core parts.

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