Simplified Model of VVER–1000 Thermal Hydraulic Process

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Abstract. This report introduce developed mathematical model of thermal hydraulic process which occurs in the core of VVER – 1000 type of nuclear reactor and the coolant flow is considered in one dimension. Navier – Stokes differential equations system is taken like basis – namely continuity equation and momentum and energy conservation equations with two algebraic equations of state by which closure relationship is done. Following the approach of simplifying the model in momentum and energy equations some simplifying assumptions is made like Reynolds term in momentum equation is neglected and diffusion term – in energy equation. The differential equations system can be split into two parts: at one hand continuity equation and momentum equation are solved regarding velocity and pressure distribution and at another – solving energy equation. This report is considering the second case – solving the unsteady energy equation at prescribed distributions of velocity and pressure. By using one of the algebraic state equation, $i = c_v T$, the energy equation is written regarding the temperature. Velocity and pressure given in the model are estimated by the thermal hydraulics means. The energy equation is solved by finite volumes method at which considered region is divided by N finite volumes, scalar values T and P are represented at central points at volumes and the velocity – at borders of these volumes by so called staggered grid. The equation is integrated at the boundaries of every finite volume and in time at the interval $[t,\ t + \Delta t]$. For results obtained by integration is applied Crank-Nicolson semi-implicit scheme. As a result this gives an algebraic system with three diagonal matrix which can be solved with Crout effective algorithm.

Keywords: finite volumes method, RELAP5, thermal hydraulics

1 Introduction

It is known that there are a number of computer codes which are carrying out calculations of thermal-hydraulic processes occurring in nuclear reactors. One of these codes is RELAP5 which is developed for best estimate simulation of light water reactor cooling system during postulated accidents [2]. The programs' main issue is that they are with closed source and the user can't make any changes in models and computation procedures. Therefore sometimes it is useful to develop one's own code.

That report considers the development of such code in MATLAB environment. It presents one dimensional flow of water coolant at WWER – 1000 nuclear reactor's core. As a basis for creating the model is used Navier-Stokes differential equation system, namely – conservation of mass equation, momentum and energy conservation equations with two additional thermodynamic algebraic equations. Following the approach which is used in RELAP5 [2] some terms are neglected: in equation of momentum – Reynolds term and in equation of energy – diffusion term. The system that governs the process is

$$\begin{aligned} &\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho\, u) = 0 \\ &\frac{\partial(\rho\, u)}{\partial t} + \frac{\partial(\rho\, u\,.\,u)}{\partial x} = -\frac{\partial P}{\partial x} \\ &\frac{\partial(\rho\, i)}{\partial t} + \frac{\partial(\rho\, i\,.\,u)}{\partial x} = -P\frac{\partial u}{\partial x} + Q \end{aligned} \qquad (1)$$

$$P = P(\rho, T) \qquad \text{and} \qquad i = c_v T\,. \qquad (2)$$

The system can be split into two and be resolved in two parts:

- solving first two equation regarding velocity and pressure distribution;
- solving energy equation regarding temperature.

In this report the second part is taken into account, namely – solving energy equation regarding temperature distribution at given values of velocity and pressure.

By using the second equation of state from Eq. (2) energy conservation equation can be written regarding only the unknown T

$$\frac{\partial}{\partial t}\left(\rho\, c_v\, T\right) + \frac{\partial}{\partial x}\left(\rho\, c_v\, T\, u\right) = -P\frac{\partial u}{\partial x} + Q\,, \qquad (3)$$

where c_v is thermal conductivity at constant volume.

This equation should be solved at some initial and boundary conditions

- initial condition $T(x, 0) = T_0(x)$ corresponds to core temperature at initial moment $t = 0$;
- boundary conditions

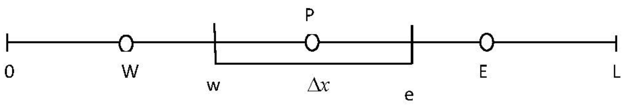
  $$T(0, t) = T_b \quad \text{and} \quad \frac{\partial T}{\partial n}\left(L, t\right) = 0$$

  correspond to the inlet temperature of the fluid at the beginning of the core and lack of heat flux at the end of the core.

2 Numerical Method

To solve the problem the finite volume method is used.

At this method core's height is L; split the interval $[0, L]$ (represented for convenience on the axis Ox) to N finite volumes:



Consider one representative finite volume (with length Δx) in the range between (w) and (e) points with central point P. Neighboring volumes has central points respectively marked as W and E.

Let's integrate Eq. (3) into the finite volume range on the axis x and in the interval $[t, t + \Delta t]$.

$$\int_t^{t+\Delta t} \left[\int_{c_v} \frac{\partial}{\partial t}(\rho c_v T) dx \right] dt + \int_t^{t+\Delta t} \left[\int_{c_v} \frac{\partial}{\partial x}(\rho c_v T u) dx \right] dt = = - \int_t^{t+\Delta t} \left[\int_{c_v} P \frac{\partial u}{\partial x} dx \right] dt + \int_t^{t+\Delta t} \left[\int_{c_v} Q \, dx \right] dt \quad (4)$$

The integration is made like this – if the integrand function is a derivative with respect to the integrating variable then Newton-Leibnitz formula is used, otherwise is used mean value theorem for integrals.

If change the order integration of at first term of Eq. (4), we will get

$$\int_{c_v} \left[\int_t^{t+\Delta t} \rho c_v \frac{\partial T}{\partial t} dt \right] dx = \rho \, c_v \left(T_p^{n+1} - T_p^n \right) \Delta x \quad (5)$$

For the second term on the left side of Eq. (4) we will have:

$$\int_t^{t+\Delta t} \left[\int_{c_v} \frac{\partial}{\partial x} (\rho \, c_v \, T u) \, dx \right] dt = = \int_t^{t+\Delta t} [\rho \, c_v \, T_e \, u_e - \rho \, c_v \, T_w \, u_w] = = \int_t^{t+\Delta t} \left[\rho \, c_v \, \frac{T_E + T_P}{2} u_e - \rho \, c_v \, \frac{T_W + T_P}{2} u_w \right] dt \quad (6)$$

Integrals with respect to t containing T_P, T_W, T_E can be represented as (example for T_P)

$$\int_t^{t+\Delta t} T_P dt = [\theta \, T_P^{n+1} + (1-\theta) T_P^n] \Delta t \,. \quad (7)$$

The parameter θ takes values from 0 to 1. The 0 value means using fully explicit scheme in the process of T_P^{n+1} searching, while at $\theta = 1$ leads to fully implicit scheme which requires system of algebraic equations to be solved. In the present article is accepted value $\theta = 1/2$ at first which gives Crank-Nicolson semi explicit scheme.

The right side integration gives the following expression:

$$[-p(u_e - u_w) + \overline{Q} \Delta x] \Delta t \,. \quad (8)$$

Here it may be noted that while scalar values ρ, T, P are represented at central points of volumes, the velocities are represented at their boundary points. Such grid is called shifted (*staggered grid*) and it is applied to avoid some unphysical oscillations in solution.

If the formulas from Eq. (5) to Eq. (8) are arranged by unknowns T_P^{n+1}, T_W^{n+1}, T_E^{n+1}, we will obtain equation of the type

$$A_W T_W^{n+1} + A_P T_P^{n+1} + A_E T_E^{n+1} = -P(u_e - u_w) + + \overline{Q} \Delta x - A_W^0 T_W^n - A_P^0 T_P^n - A_E^0 T_E^n \,, \quad (9)$$

where:

$$\begin{aligned} A_P &= \rho c_v \frac{\delta x}{\Delta t} + \frac{1}{2} \rho \, c_v (u_e - u_w) \theta \,, \\ A_P^0 &= -\rho \, c_v \frac{\Delta x}{\Delta t} + \frac{1}{2} \rho \, c_v (u_e - u_w)(1-\theta) \,, \\ A_W &= -\frac{1}{2} \rho \, c_v \, u_w \theta \,, \\ A_W^0 &= -\frac{1}{2} \rho \, c_v \, u_w (1-\theta) \,, \\ A_E &= \frac{1}{2} \rho \, c_v \, u_e \theta \,, \\ A_E^0 &= \frac{1}{2} \rho \, c_v \, u_e (1-\theta) \,. \end{aligned} \quad (10)$$

The source term Q is calculated by the formula $Q = \alpha(T_Q - T_P)$, where α is the convective heat exchange coefficient and T_Q is the fuel column temperature.

Equation (9) can be written for central points of all of the internal volumes. For boundary volumes are written non-standard equations which reflect boundary conditions.

The matrix on the resulting system of linear algebraic equations has three-diagonal structure. To solve the system the Crout effective algorithm can be used with computation complexity $O(N)$.

3 Computational Experiments

The described numerical method is realised at MATLAB environment. Numerical experiments are made with the following values of the parameters – core's height $L = 4.57$ m, active height (fuel column height) $L_q = 3.55$ m, water thermal conductivity at 300°C $c_v = 5.65$ kJ/kg, convective heat exchange coefficient $\alpha = 3$ kJ/cm^{2}K, finite volumes number $N = 50$, water temperature at the core's inlet $T_b = 290$°C.

Essential part for solution's stability has the time step Δt. Computations are made with three integration schemes with parameter values $\theta = 0, \, 0.5, \, 1$, namely – explicit, semi implicit and fully implicit schemes. All of the three led to wrong and unstable result at time step $\Delta t = 0.01$ (See Figure 1).

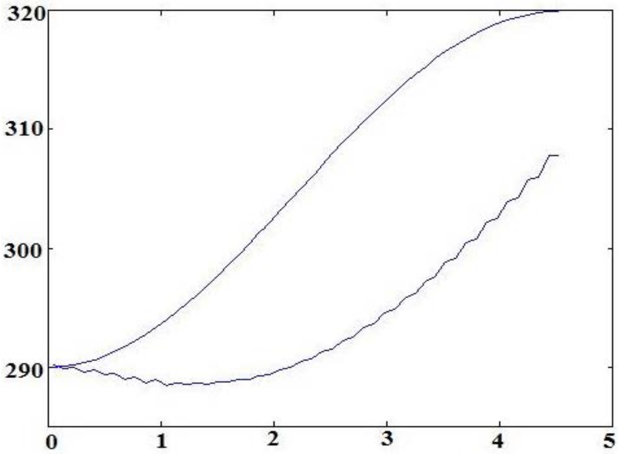


Figure 1. Unstable solution at time step equal to $\Delta t = 0.01$.

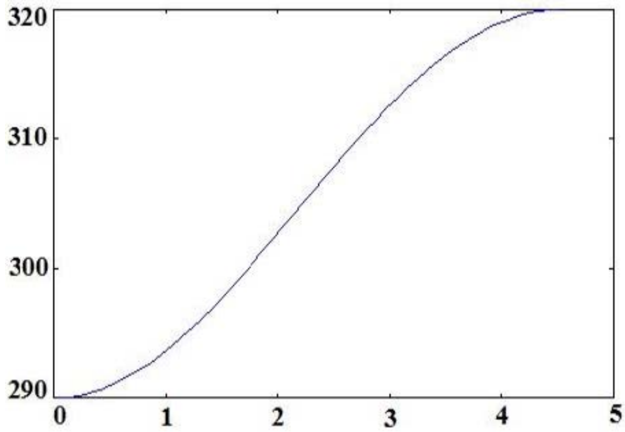


Figure 2. Stable solution at time step equal to $\Delta t = 0.00001$.

When the time step is $\Delta t = 0.00001$ the solution reaches steady-state without oscillations (see Figure 2).

4 Conclusions

This report is an attempt to implement finite volume method for solving one complex problem. In this problem three schemes have been testes for solving the unsteady problem.

In a further study diffusion term can be taken into account as a mechanism for carrying out the process of heat transfer.

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[2] RELAP5/MOD3.3 CODE MANUAL VOLUME I: CODE STRUCTURE, SYSTEM MODELS, AND SOLUTION METHODS