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Distortion of the Neutron Flux Profile in Reactor Core Induced by Control Rods

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Abstract. The method designed to calculation the axial profile of neutron flux in reactor core based on diffusion theory are presented. The core model describes two zones – upper part containing control rods (additional absorber) and unrodded bottom part of core. The solution of the system of diffusion equations is obtained using the criticality condition.

The change of the amplitude and shape of the axial profile of neutron flux at varied withdrawal of control rods are calculated. An estimation of the non-uniformity of heat release rely on the elevation of core has been obtained and a comparison with the acceptable value of axial offset has been made.

Keywords: control rods, neutron flux, diffusion model, axial offset, reactor core.

1 Introduction

To ensure reliable and safe operation of the nuclear reactor, the thermal power of reactor core is continuous monitored. In addition, it is desirable that the heat release be maximally evenly distributed over the core volume. Even when the total thermal power is supported on permissible range the occurrence of the local power peaks may lead to go beyond the safety margins of operating reactor.

No uniformity of heat generating in reactor core is due to many factors.The main source of heat generation is a nuclear fission reaction. The number of fission acts depends on the concentration of nuclear fuel atoms and neutrons that can cause fission.

The physics of the reactors determines that the neutron flux can't be uniform over core volume due to neutrons diffusion. In view of the escape of neutrons from the core in its center, the neutron concentration is always higher than at the periphery. The neutron reflector makes it possible to reduce the neutron leakage and to reduce the nonuniformity of the neutron flux, but only partially.

The local concentration of the fission fuel nuclei changes during the reactor campaign as a result of the processes of burnout and the accumulation of secondary nuclear fuel.

The presence of materials absorbing neutrons has a significant effect on the heat generation field in the core. The reactor reactivity is controlled by means of control rods and chemical shim (boric acid – H_3BO_3 in primary circuit coolant). The concentration of boric acid is highest at the beginning of the reactor campaign and reduced to almost zero at the end of reactor campaign.

All of the factors listed above, excluding the control rods, do not disturb the symmetry of the power field in the reactor core. To provide the control function in PWR and VVER, the control rods are inserted from above by 10 ... 20% of the height of core. Due to the presence of a strong absorber, the neutron flux is eaten in the upper part of the core and the peak of power in the core is shifted downward, thus distorting the neutron flux profile and heat generation along the height of reactor core.

Figure 1 shows the placement of fuel assemblies in the core of the VVER-1000 reactor. In each fuel assemble can be inserted 18 control rods which are form a cluster. There are ten groups of control rods. Only a tenth group of control rods are used to control the reactor at nominal level of power (see Figure 1). Other groups of control rods used at emergency shutdown of a nuclear reactor. The control rods belonging to the same group are distributed evenly over the core, which does not distort the symmetry of the radial profile of the neutron flux.

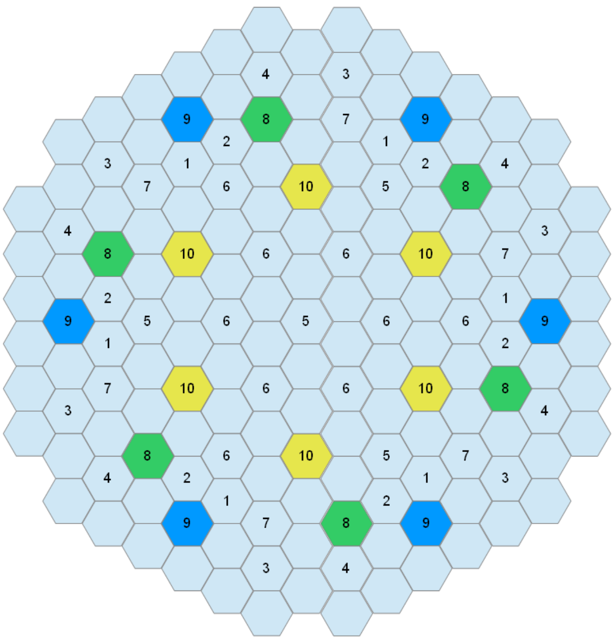

Figure 1. Groups of control rods in core of VVER 1000.

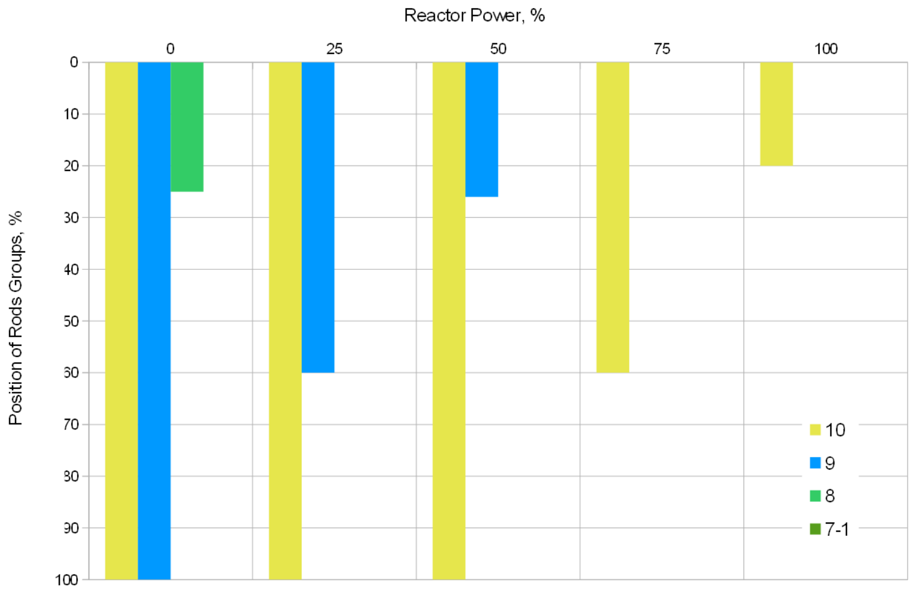

Figure 2. Positions of control rods groups at definite reactor power [1].

The position of the control rod groups on the height of reactor core depends on the level of reactor power and, as an example, the position of the control rod groups at the beginning of core life is shown in Figure 2 according to the data of [1]. Evidently, the axial profile of the neutron flux will be significantly distorted at any reactor power due to the presence of control rods in the reactor core.

2 General Approach to Calculation of the Neutron Flux Profile in the Reactor Core

The problem of calculating the neutron flux in the region of the reactor core of various geometries and taking into account the neutron reflector are considered in detail [2–5]. However, the problem of calculating the asymmetric neutron flux profile when additional absorber is in a part of the core has not been studied enough. To solve it, the approaches and methods used to calculate the core, in which there are several areas, for example, with various enrichment of fuel, or the core with reflector, can be used. However, in all these problems the solution is in the form of a symmetric neutron flux field.

The reactor core of VVER has a cylindrical form and the neutron flax in the volume of core is a function of two variables: radius r and height z. Assuming the possibility of separating the variables, we get

$$\Phi(r,z) = Z(z)Y(r)\,. \tag{1}$$

The axial $Z(z)$ and radial $Y(r)$ profile of neutron flax depends on the geometrical buckling B^2 and is a solution of equations

$$\Delta Z(z) + B_z^2 Z(z) = 0\,, \tag{2}$$
$$\Delta Y(r) + B_r^2 Y(r) = 0\,, \tag{3}$$
$$B^2 = B_z^2 + B_r^2 = (\pi/H)^2 + (2.405/R)^2 \tag{4}$$

and the general solution for a cylindrical reactor has the form

$$\Phi(r,z) = \sin\left(\frac{\pi z}{H}\right) J_0\left(\frac{2.405 r}{R}\right), \tag{5}$$

where H and R are the dimensions of reactor core, and z, r are the current coordinate.

It should be emphasized that this solution provides for identical conditions on the boundaries of the core and is symmetrical with respect to the vertical axis and the center of the height of reactor core.

3 Calculation of Axial Neutron Flux Profile Taking into Account the Position of Control Rods

To investigate the influence of the displacement of a group of control rods on the distribution of power generation in reactor core, we consider a simple model of a cylindrical reactor in a single-group diffusion approximation [5]. The core is divided into two zones: the upper part containing absorbing rods; and the lower part – without rods. Changes in the position of the group of control rods can be represented as a shift of the boundary between these parts of the core.

For this model two equations similar to equation (2) can be written. We denote the axial flow profile in the lower part of the core $Z_1(z)$ and in the upper part with absorbing rods $Z_2(z)$.

$$\Delta Z_1(z) + B_{1z}^2 Z_1(z) = 0\,, \tag{6}$$
$$\Delta Z_2(z) + B_{2z}^2 Z_2(z) = 0\,. \tag{7}$$

It should be noted that B_{2z}^2 is negative, because for the upper part, where additional absorbers are located, the multiplication factor is $K_\infty < 1$.

The general solutions of equations (6), (7) have the form

$$Z_1(z) = A_1 \sin(B_{1z} z)\,, \qquad z = 0 \ldots h\,, \tag{8}$$
$$Z_2(z) = A_2 \sinh\left(B_{2z}[H - z]\right), \qquad z = h \ldots H\,, \tag{9}$$

where h is the current position of control rods.

The integration constants A_1 and A_2 are determined by applying the boundary conditions about the equality of the neutron flux $F(x)$ and the current of the neutrons $J(x)$

on the boundary between two zones at $z = h$. Using Fick's law $J(x) = -D\,\mathrm{d}F(x)/\mathrm{d}x$, we obtain

$$Z_1(h) = Z_2(h)\,, \tag{10}$$

$$D_1 \frac{\mathrm{d}Z_1(h)}{\mathrm{d}z} = D_2 \frac{\mathrm{d}Z_2(h)}{\mathrm{d}z}\,. \tag{11}$$

Taking into account that the values of the diffusion coefficients for all materials of core are close in magnitude, we can assume that $D_1 = D_2$. After substituting expressions (8), (9) and dividing the right and left sides of equations (10), (11), we obtain the criticality condition

$$\frac{1}{B_{1z}} \tan\left(B_{1z} h\right) = -\frac{1}{B_{2z}} \tanh\left(B_{2z}\left[H-h\right]\right). \tag{12}$$

The criticality equation is transcendental and the value of B^*_{1z} that we search can be found by an iterative method. Using the expression for the geometrical buckling $B^2_{2z} = (\pi/[H-h])^2$ and introducing the variable $x = B^*_{1z} h$, we transform (12) to the form

$$\tan(x)/x = -\frac{[H-h]}{\pi h} \tanh(\pi)\,. \tag{13}$$

The relationship between the constants A_1 and A_2 from equation (10) is

$$\frac{A_2}{A_1} = \frac{\sin\left(B_{1z} h\right)}{\sinh\left(B_{2z}[H-h]\right)} = \frac{\sin(x)}{\sinh\left(B_{2z}[H-h]\right)}\,. \tag{14}$$

The graphical solution of the criticality equation is presented in Figure 3, where the horizontal line RSE – the right side of equation (13) and its intersection with the function $\tan(x)/x$ gives the value x^*, which provides the reactor criticality. Exact solution was found in the calculation as a result of successive iterations.

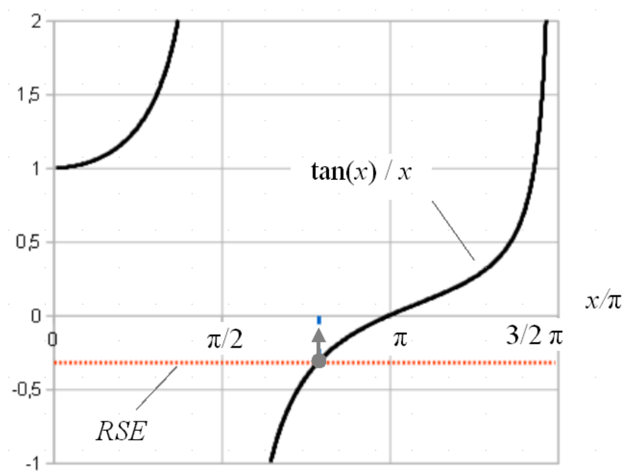

Figure 3. Solution of the criticality equation (at $h = H/2$, $x = 0.7881\pi$).

Independent solutions for the two parts of core corresponding to equations (8), (9) are shown in Figure 3. The values of the functions $Z_1(z)$, $Z_2(z)$ and the height coordinate of core z are normalized by one. The joint solution under the criticality condition is represented by line 3 in Figure 4.

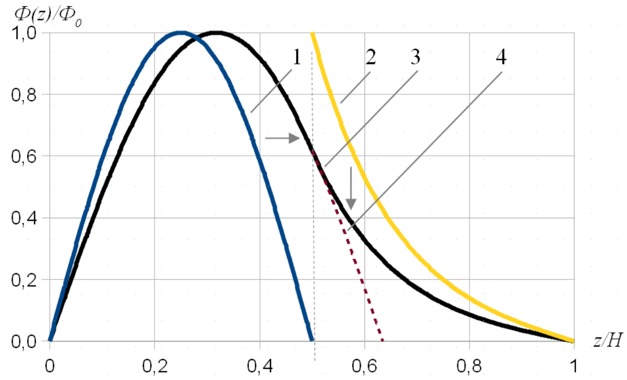

Figure 4. Axial profile of the neutron flux at criticality ($h = 0.5H$): $Z_1(z) = A_1 \sin(B_{1z} z)$ (1); $Z_2(z) = A_2 \sinh(B_{2z}[H - z])$ (2); joint solution (3); extrapolation (4).

The correctness of founded solution is confirmed by the continuity and smoothness of the function on the boundary of the core parts at $z = h$. The superposition of the criticality condition leads to a scaling of the function $Z_1(z)$ along the z axis and the function $Z_2(z)$ in amplitude. The dashed line (4) is an extrapolation of the function $Z_1(z)$ beyond the lower part of the core. The maximum of the axial flux profile is shifted up.

It is important to note that in the layer above and below the coordinate in which the maximum of the neutron flux profile is located, the core composition is same. Therefore, the profile should be symmetrical with respect to its maximum within the homogeneous layer.

The results of calculation the axial profile of the neutron flux at certain position of control rods are shown in Figure 5. To estimate the reactor safety, it is important not only to know the position of the neutron flux peak, but also its magnitude. The amplitude of the neutron flux profiles shown in Figure 5 is normalized to unity that makes it impossible to see changes in the magnitude of the maximum for different positions of the control rods.

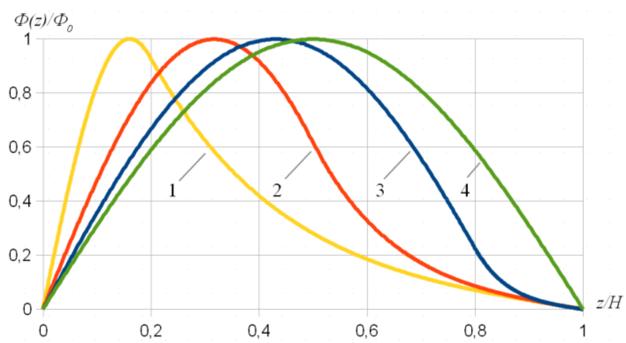

Figure 5. Axial profile of neutron flux depending on the control rods withdrawal h/H: 0.2 (1); 0.5 (2); 0.8 (3); 1.0 (4).

If we assume that the average value of the neutron flux remains unchanged, then one can obtain the scaling factor of the profile amplitude $\alpha(h)$. As the base value of neutron flux Φ_0 can be chosen its integral mean value with fully withdrawing of control rods. The integral mean values of the neutron flux of two parts of reactor core are defined as the integrals of the functions $Z_1(z)$ and $Z_2(z)$

$$\begin{aligned} I_1(h) &= \int_0^h Z_1(z)dz \\ &= -A_1/B_1 \cos(B_1 z)\Big|_0^h , \end{aligned} \tag{15}$$

$$\begin{aligned} I_2(h) &= \int_0^h Z_2(z)dz \\ &= -A_2/B_2 \cosh(B_2[H-z])\Big|_h^H , \end{aligned} \tag{16}$$

and after substituting the limits of integration, we obtain

$$I_1(h) = A_1 h \frac{1-\cos(x)}{x} , \tag{17}$$

$$I_2(h) = A_2[H-h]\frac{1-\cosh(\pi)}{\pi} . \tag{18}$$

The base value of Φ_0 is obtained from the expression (17) under the condition that $x = \pi$

$$\bar{\Phi}_0 = \frac{I_1(h=H)}{H} = \frac{2}{\pi} . \tag{19}$$

The average value of the neutron flux, when the function is normalized by unity, is

$$\begin{aligned} \bar{\Phi}(h) = \frac{I_1(h)+I_2(h)}{H} = \frac{1}{H}\Big\{ A_1 h \frac{1-\cos(x)}{x} \\ + A_2[H-h]\frac{1-\cosh(\pi)}{\pi} \Big\} \end{aligned} \tag{20}$$

and scaling factor of the profile amplitude is

$$\begin{aligned} \alpha(h) = \frac{\bar{\Phi}(h)}{\bar{\Phi}_0} = \frac{\pi}{2H}\Big\{ A_1 h \frac{1-\cos(x)}{x} \\ + A_2[H-h]\frac{1-\cosh(\pi)}{\pi} \Big\}. \end{aligned} \tag{21}$$

The calculated values of the scaling factor $\alpha(h)$ are given in the Table 1.

Taking into account the calculated scaling factors, the axial profile of the neutron flux for certain control rods withdrawal is shown in Figure 6. The displacement of the control rods causes a significant distortion of the axial profile of the neutron flux and leads to an increase in its peak value. For example, at $h/H = 0.2$, the peak value of the neutron flux exceeds the average value by 1.78 times, which leads to a proportional increase of the local power on the elevation where peak located. Thus, the movement of the control rods can cause significant increase of the local power generation, so the interval of permissible positions of the control rods should be limited.

Table 1. Scaling factor of the neutron flux amplitude $\alpha(h/H)$

h/H	$\alpha(h/H)$	h/H	$\alpha(h/H)$	h/H	$\alpha(h/H)$
1	1.000	0.6	1.316	0.3	1.639
0.9	1.071	0.5	1.412	0.2	1.782
0.8	1.147	0.4	1.518	0.1	1.957
0.7	1.228				

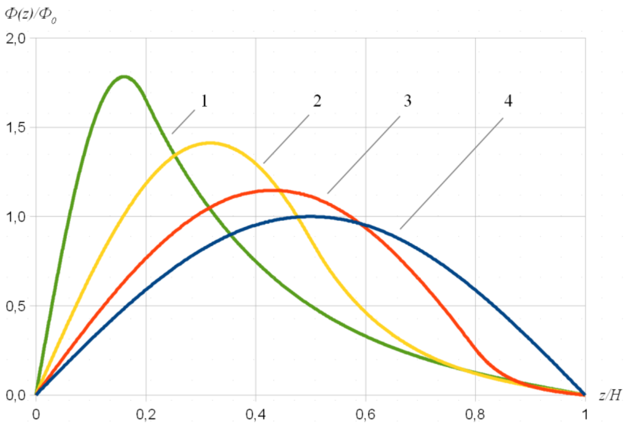

Figure 6. Changing the amplitude and shape of the axial profile of neutron flux depending on the control rods withdrawal h/H: 0.2 (1); 0.5 (2); 0.8 (3); 1.0 (4).

4 Axial Offset Evaluation

With a symmetrical heat generation field in reactor the maximum parameters at the core center or the unevenness coefficients, which are defined as the ratio of the peak to average value of the neutron flux are a representative indicators of the axial peaking factors. With asymmetrical heat generation field, it is important not only value of neutron flux peak, but its location, which depends on the control rods position. Therefore, an additional indicator is introduced – axial offset (AO), which is defined as the relative difference in heat generation in the top and bottom halves of reactor core, expressed as a percentage of the total heat generation in the core.

$$\mathrm{AO} = \frac{Q_1 - Q_2}{Q_1 + Q_2} , \tag{22}$$

where Q_1 and Q_2 are the heat generation in the top and bottom halves of reactor core, respectively.

To calculate the AO value is sufficient to know the axial profile of the neutron flux if the enrichment of nuclear fuel are same over the entire height of reactor core, which is only valid for fresh fuel. Then AO can be calculated from the value of the mean integral flux I_{top} and I_{bottom} in the lower and upper halves of the reactor core

$$\mathrm{AO} = \frac{I_{\text{top}} - I_{\text{bottom}}}{I_{\text{top}} + I_{\text{bottom}}} , \tag{23}$$

$$I_{\text{bottom}} = \int_0^{H/2} Z(z)\mathrm{d}z , \quad I_{\text{top}} = \int_{H/2}^{H} Z(z)\mathrm{d}z . \tag{24}$$

To find the AO, one can use the axial profile of the neutron flux, which are normalized to unity or withtaking into account the scaling factor $\alpha(h)$. In both cases, the calculated AO value will be the same, since the AO is not sensitive to the change in the amplitude of the neutron flux profile.

The calculated value of AO for different positions of the control rods is shown in Figure 7. It is important to note that in the position range of the control rods $h/H = 0.5 \ldots 1.0$, the OA value is proportional to the shift of the control rods. This is due to the fact that in this range

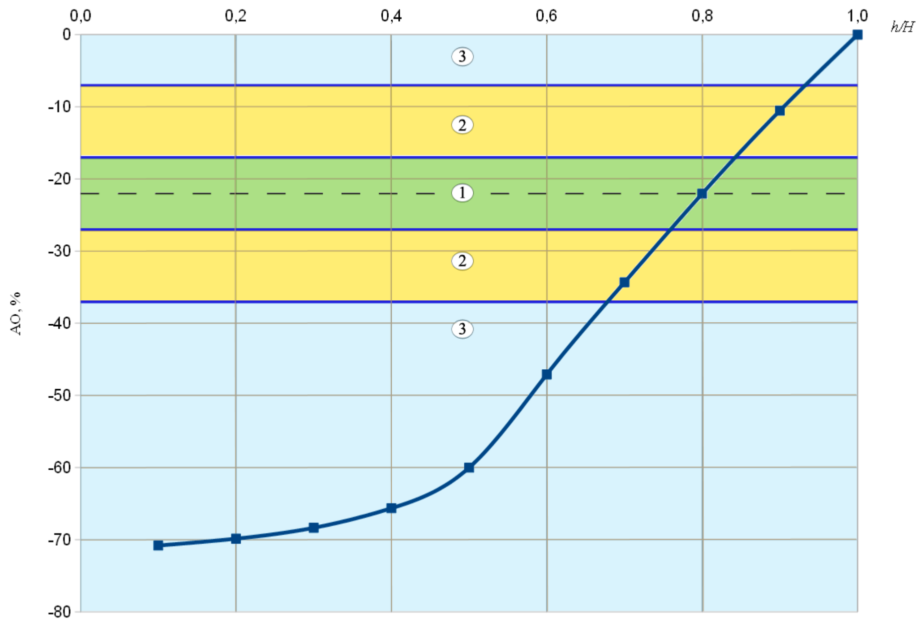

Figure 7. AO depending on the control rods withdrawal. Range of AO: recommended (1); allowable (2); unallowable (3).

the profile of the neutron flux in the bottom half of the reactor core is described by one function and the change in the control rods position leads to a linear change in its integral mean value I_{bottom}.

When the control rods are inserted below $h/H = 0.5$, the distortion of the profile slows down and the AO value asymptotically approaches a value of about 70%, but this value is not acceptable with the reactor safety point of view.

The nominal values of AO that must be maintained during the reactor operation depend on the moment of the reactor campaign, and the permissible deviations are regulated taking into account the reactor power and time of the AO deviation from the nominal value. According to technological regulations [6], the permissible deviation of AO at the nominal power level of the reactor should not exceed $\pm 15\%$, from its nominal value.

Using Figure 7 the evaluation of the permissible range of the control rods position can be done. For example, if we assume that reactor is operating on power of 100% and the nominal position of the tenth group of control rods is a withdrawal $h/H = 0.8$ (inserted by 20%) then nominal value of AO equal 22%. In addition we can note that AO stays in the allowable range if control rods inserted by $7 \ldots 37\%$, but no more and no less than this.

However, it should be borne in mind that even if the distortion caused by the control rods is absent, the AO value will not be zero. The heating of water in VVER core causes a slight decrease in reactivity along the altitude of the reactor core and the AO value dope down by $5 \ldots 10\%$ [7]. During reactor campaign this factor is compensated by a more intensive burnout of fuel in the bottom half of reactor core. Thus, when carrying out a more detailed calculation of the axial profile, it is necessary to take into account the change of the thermodynamic properties of coolant and the composition of nuclear fuel over height of reactor core.

5 Conclusion

At any position of control rods in reactor core a distortion of the axial profile of the neutron flux and heat generation fields are appears. The local peak of the axial flux distribution can almost 2 times exceed the average flux value and the AO may reach – 70%.

The AO value stays in the allowable range at nominal power of reactor if control rods are inserted on more than one third of the height of reactor core.

On the basis of the presented calculation method, an improved assessment of the axial profile of heat generation in reactor core can be obtained, if take into account the change of the thermodynamic properties of coolant and the composition of nuclear fuel over height of reactor core.

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